

Spatiotemporal chaos control with a target wave in the complex Ginzburg-Landau equation system

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An effective method for controlling spiral turbulence in spatially extended systems is realized by introducing a spatially localized inhomogeneity into a two-dimensional system described by the complex Ginzburg-Landau equation. Our numerical simulations show that with the introduction of the inhomogeneity, a target wave can be produced, which will sweep all spiral defects out of the boundary of the system. The effects exist in certain parameter regions where the spiral waves are absolutely unstable. A theoretical explanation is given to reveal the underlying mechanism.

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I. INTRODUCTION

Controlling deterministic chaos has become an active field in the study of nonlinear dynamics over the past few decades. Since the pioneering work of Ott, Grebogi, and Yorke [1], significant progress [2–5] has been achieved in controlling chaos in systems with few degrees of freedom. These efforts have been naturally extended to control spatiotemporal chaos [6] in spatially extended systems, because the great potential of applications exists in plasma devices [7], laser systems [8], chemical reactions [9], and biological systems [10], where both spatial and temporal dependence need to be considered.

Theories about spatiotemporal chaos have been extensively studied in the complex Ginzburg-Landau equation (CGLE) system [11], which describes universal dynamics features of spatially extended systems near a supercritical Hopf bifurcation. It exhibits defected mediated turbulence or spiral turbulence in a wide parameter region. The idea behind the previous attempts to achieve the turbulence control is to trace and stabilize one previously unstable wave-generating defect by injecting weak perturbations near the defect core. A stable spiral wave is thus developed to cover all the uncontrolled region along the wave propagation direction. For example, Aranson *et al.* [12] suggested a method of turbulence control by applying around a unstable defect a localized feedback injection with a time delay. Zhang *et al.* [13] achieved the control by generating a spiral wave seed, and growing this seed into a stable spiral by injecting a local periodic signal around it.

In this paper we will propose another method of spiral turbulence control by introducing a localized inhomogeneity at a random location in the system described by two-dimensional (2D) CGLE. A target wave will be produced around the inhomogeneity. In certain parameter regions the target wave is stable while spiral waves are absolutely unstable, and the target wave drives the spiral turbulence out of the system. The method is surprisingly simple and highly efficient.

In the next section, we perform a preliminary analysis of the CGLE, describe parameter regions where different stabilities have been observed, and report our numerical results for the turbulence control. In Sec. III, we analyze the reason why the target wave can be generated, stable, and dominative when competing with surrounding spatiotemporal chaos. We give the conclusion of our work in Sec. IV.

II. MODEL AND SIMULATION

The homogeneous CGLE describes spatially extended media in which the homogeneous state is oscillatory and the system is near a supercritical Hopf bifurcation. It has the form

$$\frac{\partial A}{\partial t} = A - (1 + ic)|A|^2 A + (1 + ib)\nabla^2 A, \quad (1)$$

where b, c are real control parameters, $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$, and $A(\mathbf{r}, t)$ is the complex variable. A steadily rotating spiral solution of Eq. (1) has the general form

$$A(\mathbf{r}, t) = F(\mathbf{r}) \exp\{i[\sigma\theta + \psi(r) - \omega t]\}. \quad (2)$$

For large r the spiral wave asymptotes to a plane wave with the wave number $[k = (d\psi/dr)]_{r \rightarrow \infty}$, which is independent of r . Substituting the constant amplitude plane wave solution $A = \sqrt{1 - k^2} \exp(ikr - i\omega t)$ into Eq. (1), one can get the following dispersion relation:

$$\omega = c + (b - c)k^2. \quad (3)$$

If one ignores the curvature effect, a target wave solution can be considered as a plane wave solution, so that the dispersion relation of Eq. (3) is approximately valid to target wave solutions.

In the following discussion we fix $b = -1.4$. Figure 1 is a cut of the phase diagram in the $b - c - k$ parameter space. It shows several regions where the plane-wave solution has different stabilities [11]. The convectively unstable region is due to Eckhaus instability, where the traveling waves of certain wave numbers can remain stable in the convective sense, because the growth rate of a perturbation is smaller than the traveling speed of the waves; while in the absolutely unstable

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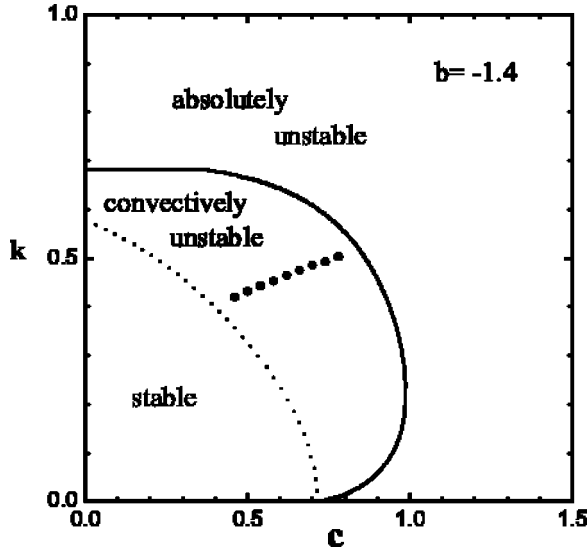


FIG. 1. Regions with different stabilities in the k - c plane with $b = -1.4$. The solid line and dotted line are the onset of absolute instability and convective instability, respectively. The wave numbers of the spiral waves (dark dots) are uniquely decided by the parameters b and c .

region, the perturbation growth rate becomes larger than the wave speed. In this case the system will quickly fall into a state of defect-mediated turbulence. For a spiral wave solution, the wave number k is uniquely determined by the parameters b and c , as shown in Fig. 1. Thus, with a fixed value $b = -1.4$, there exist a critical value $c_0 = 0.8$ beyond which the system undergoes the transition from ordered spiral waves to spiral turbulence. Our study focuses on controlling spiral turbulence in the absolutely unstable region.

In all numerical simulations the space variables are discretized to 256×256 sites under no-flux boundary conditions. We start with random initial conditions with parameter $c = 0.9$, which is in the absolutely unstable region, as shown in Fig. 1. We wait until the state of spiral turbulence is fully developed; an example of such a state is shown in Fig. 2(a). Then at control time $t = 0$ we introduce to the system a spatially localized inhomogeneity by changing parameter c from 0.9 to $c_I = 0.6$ in the central 5×5 sites (actually arbitrary position also works). Figure 2 demonstrates numerical simulations with the spatially localized inhomogeneity. Concentric waves are automatically emitted from the localized inhomogeneity at $t = 450$ t.u., see Fig. 2(b). The target wave is stable, and gradually invades into the region where spiral turbulence previously dominate, see Fig. 2(c). Finally after $t = 1500$ t.u., the whole system is dominated by one large target wave and the control is achieved, as shown in Fig. 2(d). Our simulation results show that for $b = -1.4$, the target wave can be generated only when $c - c_I \geq 0.3$, otherwise the target wave cannot be generated in the turbulent sea.

III. ANALYSIS

Several questions are in order. Why can the target wave be generated? Why is the target wave stable while the system

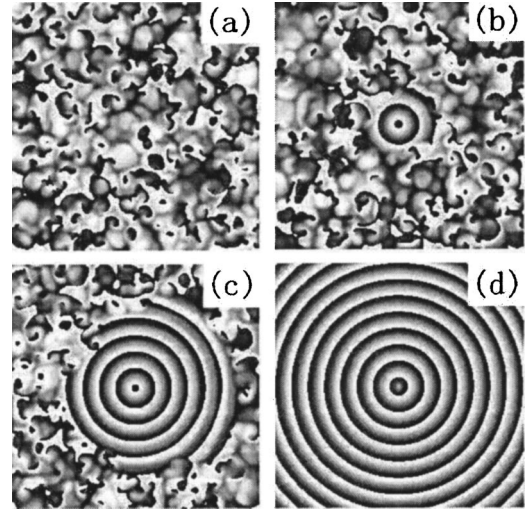


FIG. 2. Numerical simulation of the spatiotemporal chaos control with a target wave. (a) $t = 0$, while the control starts; (b) $t = 450$ t.u., the target wave start to grow; (c) $t = 1000$ t.u., the target wave almost dominates the system; (d) $t = 1500$ t.u., the whole system is occupied by a large target wave.

parameters are in the absolutely unstable region for spiral waves? And why could the target wave dominate the system when competing with around spatiotemporal chaos? In this section we try to answer these questions.

Our numerical simulations show that the target wave is the result of the introduction of localized inhomogeneity. As proved by previous experiments [14–16] as well as numerical simulations of inhomogeneous CGLE [17], the existence of target waves is attributed to local inhomogeneities which play the role of pacemakers, changing the local frequency of the bulk oscillation (ω_0) [18]. As a result, a target wave with the same frequency as the pacemaker (ω_T) is generated. The frequency change in the local area should be large enough to show the inhomogeneity, which is why in our simulations we find that the change of c should be larger than 0.3. The dynamics of the target wave is solely determined by the parameters c , b , and c_I . Inside the introduced small area the system is in the state of homogeneous oscillation with frequency $\omega_T = c_I$ [19], this determines the frequency of the target wave outside of the small area where the dispersion relation holds [see Eq. (3)]. As a result we have $c_I = c + (b - c)k^2$, which gives the wave number k of the target wave. According to this argument, the wave number of the target wave that automatically generated in the system should obey the following equation:

$$k = \sqrt{\frac{c - \omega_T}{c - b}} = \sqrt{\frac{c - c_I}{c - b}}. \quad (4)$$

As shown in Fig. 3, the results of our numerical simulation qualitatively confirm this statement. The systematic error in Fig. 3 is due to the curvature effect of the target wave. In addition, in the parameter region we focused on, i.e., $b = -1.4$, $c > 0$, $c_I > 0$, $|\omega_T| < c = |\omega_0|$, the frequency of the bulk oscillation is larger than the asymptotic frequency of the

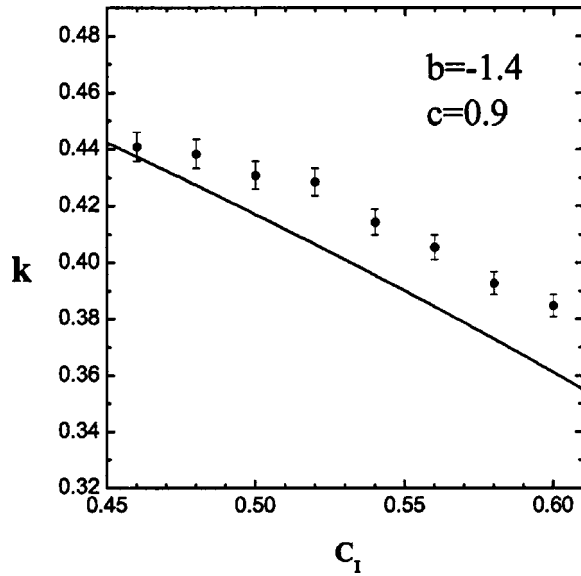


FIG. 3. The change of wave number k of the target waves that generated in the system as a function of parameter C_I . The black dots represent the simulation results; the solid line is the theoretical prediction of Eq. (4)

wave. Thus, the target wave propagates inwardly [19].

We now turn to give an explanation to the second question according to the discussion presented in the above paragraph and Eq. (3). We know that with the same parameters b and c , the value of k will decrease with the increase of the value of ω . Thus, if we can increase the frequency of a traveling wave, it is possible to drive the wave number of the traveling wave from an absolutely unstable region to a convective unstable region (see Fig. 1). According to target wave solutions, it is easy to change its frequency, because $\omega_T = c_I$ and c_I could continuously change in the interval of $[0, c - 0.3]$. That is the reason why certain target waves can be stable in the absolutely unstable region for spiral waves. The wave number of the target wave should be neither too large nor too small to keep the system behind the onset of absolute instability, so that c_I should be inside the region determined by the onset of absolute instability (solid line in Fig. 1). We display this region in the ω - c or c_I - c plane in Fig. 4, where the solid line corresponds to the onset of absolute instability for plane waves, which determines the minimum and the maximum of ω_T . The vertical dashed line ($c = 0.8$) is the onset of absolute instability for spiral waves; thus the area defined by the dashed line and the solid line in Fig. 4 is the region where a target wave could be stable but a spiral wave is unstable. The dotted line in the middle represents the existence condition for a target wave, beyond which target waves cannot be automatically generated by introducing inhomogeneity in a local region.

The mere fact that the target wave is stable does not necessarily lead to the control of spatiotemporal chaos. To achieve the control, the target wave should be able to develop, which means that the domain walls between the target wave and spiral turbulence should move outward, and gradually the target wave should dominate the whole system. In the parameter space of CGLE that we are interested, the

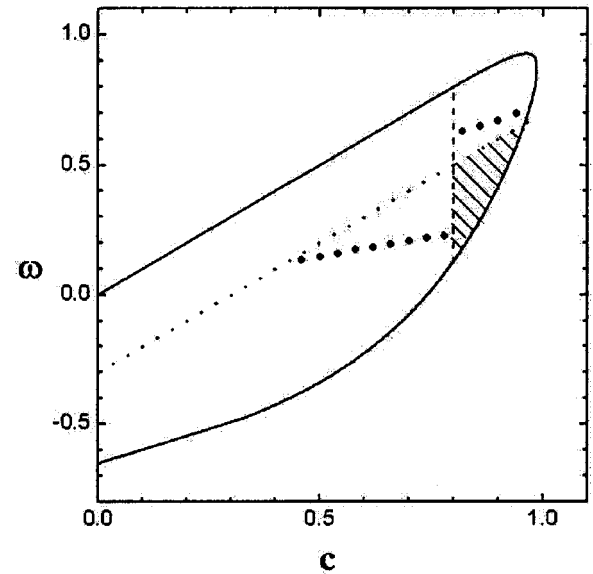


FIG. 4. Phase diagram in the ω - c plane. The solid line and dashed line show the onset of absolute instability for plane waves and for spiral waves, respectively. The dotted line in the middle represents the existence condition for target wave $c_I \leq c - 0.3$. The black dots show the change of spiral wave frequency as a function of c . A sharp increase occurs at the onset of spiral turbulence. The hatched area is the region where the control can be achieved.

movement of domain walls is a result of competition between antitarget waves and antispiral waves [19]. The request that the phase of the solution must be continuous across domain boundaries provides an equation for the velocity of the domain walls [20], which states that, for the case of $b < c$, as we have, the pattern with the lowest frequency will dominate. Spiral wave solutions in an absolutely unstable parameter region can be regarded as little spiral seeds with a very short correlation length. The black dots in Fig. 4 show the frequency of spiral wave solutions ω as a function of c . We observe a sharp increase at the onset of spiral turbulence. To have advantages of target waves over the spiral turbulence, ω_T or c_I must be smaller than the corresponding frequency of spiral wave solutions. As a result, the hatched area in Fig. 4 is the region where the spiral turbulence can be controlled by introducing an inhomogeneity and generating a target wave in the system. c_I could continuously change its value inside the region. Our simulation results with different c_I are consistent with this analysis.

IV. DISCUSSION AND CONCLUSION

The major advantage of our method of controlling defect-mediated turbulence or spiral turbulence is that it is simple, convenient, and highly efficient. There is no need to trace or lock a certain spiral defect [21]. We only need to introduce a spatially localized inhomogeneity at an arbitrary position of the system. The high efficiency is also attractive. We have tried to increase the system size to 512×512 . The change of the parameter value of the central 5×5 sites successfully brings the change of the dynamics behavior of the whole 512×512 region.

Finally, we emphasize that the control scheme can be readily applied to a reaction-diffusion system. Notice that for a reaction-diffusion system, the CGLE describes the amplitude equation near the onset of supercritical Hopf bifurcation, which satisfies $u \approx u_0 + A(R, T) \exp(i\omega_H T) + \text{c.c.}$, where u represents the concentration of a species in the system. Insert Eq. (2) into the above equation and one has $u - u_0 \propto F(r) \exp\{i[\sigma\theta + \psi(r)] + (\omega_H T/t - \omega)t\} + \text{c.c.}$ Often (not always), we have $\omega_H T/t \gg \omega$. Therefore, in a reaction-diffusion system we will observe a competition between target waves

and spiral waves which both propagate outward, and the pattern with the highest frequency ($\omega_H T/t - \omega$) will dominate.

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